**Final Project**

**Main topic and problem for the final project:**

The main purpose of the project is to introduce you how to use a computer as a research tool in an Introductory Discrete Mathematics. In this project you will be asked to find Hamilton Circuit(s) in a graph G by implementing Dijkstra’s Algorithm in any programming language.

**Hamilton Paths and Circuits:** Recall that we have developed necessary and sufficient conditions for the existence of paths and circuits that contain every edge of a multigraph exactly once. The natural question is that: Can we do the same for simple paths and circuits that contain every vertex of the graph exactly once? A simple path in a graph G that passes

through every vertex exactly once is called a Hamilton path, and a simple circuit in a graph G that passes through every vertex exactly once is called a Hamilton circuit. Hamilton paths and/or circuits can be used to solve many practical problems. Finding a Hamilton path or circuit in the appropriate graph model can solve such problems.

**Shortest-Path Problems and Algorithm:** The famous traveling salesperson problem asks for the shortest route a traveling salesperson should take to visit a set of cities. This problem reduces to finding a Hamilton circuit in a complete graph such that the total weight of its edges is as small as possible(see Section 10.6). Many problems can be modeled using graphs with weights assigned to their edges. Graph that have a number assigned to each edge are called weighted graphs. Determining a path of east length between two vertices in a graph G, let the length of a path in a weighted graph be the sum of the weights of the edges of this path.

The question is: What is a shortest path, that is, a path of least length, between two given vertices? An important problem involving weighted graphs asks for a circuit of shortest total length that visits every vertex of a complete graph exactly once. This is the famous traveling salesperson problem, which asks for an order in which a salesperson should visit each of the cities on his route exactly once so that he travels the minimum total distance.

There are several different algorithms that find a shortest path between two vertices in a weighted graph. You are asked to implement a greedy algorithm discovered by the Dutch mathematician Edsger Dijkstra in 1959. Dijkstras algorithm finds the length of a shortest path between two vertices in a connected simple undirected weighted graph. Dijkstras algorithm uses O(n2) operations (additions and comparisons) to find the length of a shortest path between two vertices in a connected simple undirected weighted graph with n vertices.

**Hamilton Circuits and Complexity:** The best algorithms known for finding a Hamilton circuit in a graph G or determining that no such circuit exists have exponential worst-case time complexity in the number of vertices of the graph G. Finding an algorithm that solves this problem with polynomial worst-case time complexity would be a nontrivial accomplishment because it has been shown that this problem is NP-complete (see Section 3.3). Thus, the existence of such an algorithm would imply that many other seemingly intractable problems could be solved using algorithms with polynomial worst-case time complexity.

***Problems for the Final Project:***

Consider the graph G of a dodecahedron, a polyhedron with 12 regular pentagons as faces, as shown in Figure 8(a) (p.699) in section 10.5. That is, the graph G = (V, E) is consists of V which is the set of the 20 vertices of the dodecahedron and E, the set of the edges of 12 regular pentagons as faces of the dodecahedron. You may put any type of weight on the graph G.

**(1)** Find a shortest path between any two given vertices in the graph G = (V, E) by implementing Dijkstra’s Algorithm in any programming language.

**(2)** Find Hamilton Circuit(s) in the graph G = (V, E) to start at any vertex of the dodecahedron and travel along the edges of the dodecahedron, visiting each of the other 19 vertices exactly once, and end back at the first vertex by implementing Dijkstra’s Algorithm (p.712) in section 10.6 in any programming language.

**(3)** Visualize the output of your implementations, that is, show the graph with the shortest path that you find between any two given vertices of G.

**(4)** Write a short paper or report that contains the source code(s) and the output of the implementations.

References

Discrete Mathematics and Its Applications, Author: Kenneth H. Rosen, Publisher: McGraw- Hill, Edition: 7, Year Published 2012.